Integral Bridges – Development of a Constitutive Soil Model for Soil Structure Interaction

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INTRODUCTION

Traditionally, engineers have used bearings and expansion joints to accommodate bridge expansion and contraction caused by daily and seasonal temperature fluctuations. Studies carried out in the late 1980s showed durability problems can be associated with bearings and expansion joints [Wallbank, 1989]. Since the mid twentieth century Integral Bridges with no expansion joints or bearings have been used. Deck expansion and contraction is accommodated by movement of the abutments into the retained fill. This eliminates the problem of durability but the movement of the abutments has been thought to cause a build up of horizontal pressures, particularly in the case of full height abutments. In the United Kingdom BA42/96 [Highways Agency, 2000] was issued and gave guidance on the soil pressures that should be adopted in design. The validity of the work on which the code of practice was based is a subject of continued debate by both researchers and practicing engineers. For this reason Integral Bridges have been used much less widely than conventional bridges.

As part of a strategy by the University of Southampton to further investigate the occuring soil pressures, Xu [2005] carried out radial controlled triaxial tests of granular material under cyclic loading. The applied strain and stress path used represented that typically experienced by an element of retained material behind an integral bridge abutment. This was the first time that the fundamental behaviour had been investigated in this way.

The further research discussed in this paper builds upon this by use of numerical modelling. The fundamental behaviour of granular material under this particular loading could not be represented by any available constitutive model and therefore a new model was be developed based on this behaviour. The basis of the model and initial validation process are discussed. The first stage of the validation process was implementation in a commercially available spreadsheet package. This was then used to develop a model in the Finite Difference Method package FLAC. Once this was implemented, the triaxial tests were modelled and the results compared to experimental data.

FUNDAMENTAL BEHAVIOUR

The basis of the developed numerical model was the work reported by Xu *et al*. [2007]. Xu [2005] carried out radial - strain controlled cyclic triaxial tests implementing a stress path typical of that for an element of soil behind an integral bridge backfilled by granular material (represented by Leighton Buzzard sand). The vertical cell pressure was kept constant to model the overburden pressure for a typical mid-height element at 4m depth. The applicable radial strain range was estimated by finite element analysis and by a geostructural mechanism [Bolton and Powrie, 1988] which ensured that the element was under typical loadings as found in service. The specimens were brought to the at rest stress state prior to cycling commencing. Various strain ranges and initial densities were considered to ensure that the results were applicable to a range of bridges with different soil conditions.

The results of Xu showed that the typical relationship between horizontal stress and radial strain was that of a hardening law. As the sample was compressed the horizontal pressure increased, and conversely upon triaxial extension the horizontal pressure returned to the active state. When this was repeated for the same radial strain range in the following cycle the maximum horizontal pressure was found to increase, as shown in Figure 1. Other key findings reported by Xu include:

- The soil densifies until a maximum value is reached, at which point it starts to dilate
- The soil stiffness, and horizontal pressures, increase regardless of whether densification or dilatation occurs
- Axial strain varies dependant on density

Xu [2005] also investigated the same situation but replacing the sand with spherical glass ballotini. Under the same loading condition no build up of horizontal pressure was found to occur. This allowed the researchers to conclude that the pressure build up was primarily due to readjustment of the soil fabric due to rolling/sliding effects of non-spherical particles close to the active state [Clayton *et al.*, 2006]. This is a significant finding as it shows the fundamental behaviour of the soil, something which previous studies had not achieved.

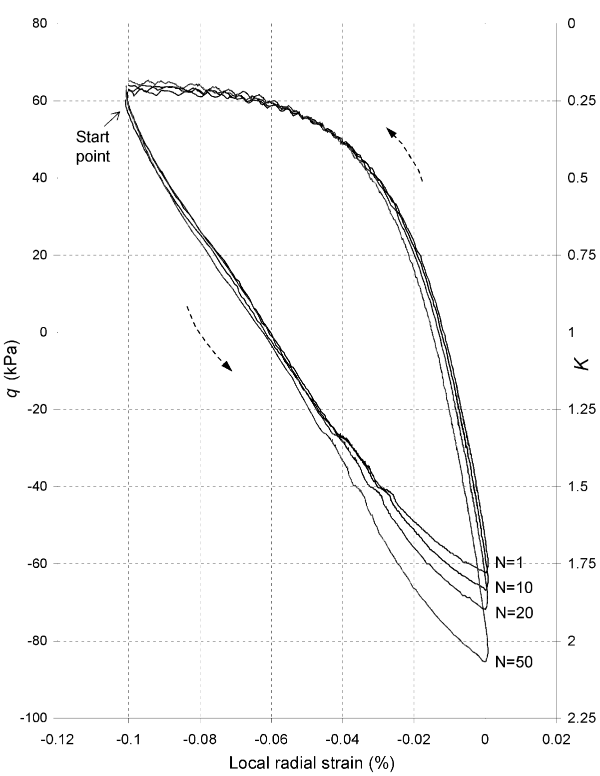

FIGURE 1 - TYPICAL CURVE OF DEVIATOR STRESS AND EARTH PRESSURE COEFFICIENT *K* AGAINST LOCAL RADIAL STRAIN FOR LEIGHTON BUZZARD SAND BY XU [2005]

BASIS OF CONSTITUTIVE MODEL

The fundamental behaviour described above was used as the basis for the model. It was noted that stiffness increased with density until dilation occurred but then continued with a similar trend post-dilation. This indicated that the influence of rolling/sliding could not be underestimated. The work of Xu [2005] had quantified the densification of the soil but this was not possible with the effects of rolling/sliding and it was not possible to separate the two mechanisms. To compensate for this the concept of a Hardening Parameter, γ, was adopted. This was a parameter which traced the density until the dilation point and then continued on the same slope beyond. This is shown in Figure 2.

As the hardening parameter relied on density it was essential to investigate the densification behaviour. It was found that the rate at which the soil densified under cycles of constant radial strain had two distinct slopes. The point at which the slope changes was designated as the Critical value of Relative Density. Therefore the change in relative density for a cycle, ΔD_r,is:

$$\Delta D_r = f(\gamma, \Delta\varepsilon_r) \tag{1}$$

where $\Delta\varepsilon_r$ is total change in radial strain; and γ the Hardening Parameter. Similarly the critical density, D_{rcrit}, was discovered to be reliant on the radial strain range.

$$D_{rcrit} = f(\Delta\varepsilon_r) \tag{2}$$

Xu used Hooke's Law to derive the secant horizontal Young's modulus, E'_h, for each cycle. This can be represented by a logarithmic curve of the form:

$$E'_h = A \ln(\Delta\varepsilon_r) + B \tag{3}$$

where A and B are functions of the Hardening Parameter, γ, and also depend on whether radial extension or compression is taking place. Similarly the relationship between radial and axial strain was found to be of the form:

$$\Delta\varepsilon_a = X(\Delta\varepsilon_r)^3 + Y(\Delta\varepsilon_r)^2 + Z(\Delta\varepsilon_r) \tag{4}$$

where X, Y and Z are functions of strain direction and the Hardening Parameter.

Poisson's ratio, υ, was required and using the experimental data this was calculated to be a constant value.

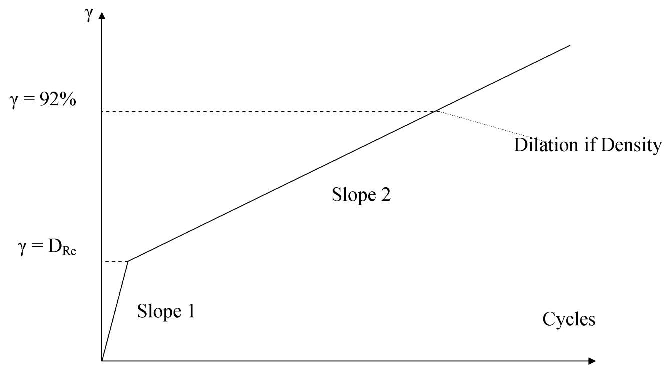

FIGURE 2 - DEVELOPMENT OF HARDENING PARAMETER, γ, AGAINST CYCLES OF RADIAL STRAIN

These relationships could then by used in conjunction with Hooke's Law to calculate the pressures developed. However, before this could be tested failure needed to be addressed. Values of the stress invariants p' and q' were plotted giving the stress path intended by Xu. The failure criterion from Critical State Soil Mechanics was considered. The failure surface of gradient M, where M is a function of the estimated angle of internal friction, was then superimposed on the plotted values. It was found from this that the soil failure coincided with the surface and therefore the decision was made to adopt this failure criterion.

IMPLEMENTATION

Upon producing the mathematical basis, implementation of the model had to be considered. The model was intended to be used in a Finite Difference Method (FDM) programme, in this case FLAC, which uses a timestep approach. This means that every cycle of radial stress change, compressive or extensive, requires to be applied over a number of timesteps. This can be represented by:

$$\Delta\varepsilon_r = \sum_1^n \delta\Delta\varepsilon_r \tag{5}$$

where $\Delta\varepsilon_r$ is total change in radial strain over the cycle; $\delta\Delta\varepsilon_r$ the step change in radial strain; and n is the number of steps considered. For example, a radial strain change cycle of 0.1% may be applied as 10 timesteps of magnitude 0.01%.

This approach was also applied to other parameters so that the equivalent Hooke's Law as shown in (6) was adopted:

$$\begin{bmatrix} \delta\Delta\varepsilon_x \\ \delta\Delta\varepsilon_y \\ \delta\Delta\varepsilon_z \end{bmatrix} = \frac{1}{E'_h}\begin{bmatrix} 1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{bmatrix}\begin{bmatrix} \delta\Delta\sigma'_x \\ \delta\Delta\sigma'_y \\ \delta\Delta\sigma'_z \end{bmatrix} \tag{6}$$

where $\delta\Delta\varepsilon_{x,y,z}$ is step change in strain; $\delta\Delta\sigma_{x,y,z}$ the step change in stress; E'_h the secant horizontal Young's modulus; and υ = Poisson's ratio.

In the case of triaxial compression and extension Hooke's Law can be reduced to:

$$\delta\Delta q = \delta\Delta\sigma_a - \delta\Delta\sigma_r = -\,\delta\Delta\sigma_r = \frac{E'_h\left(\delta\Delta\varepsilon_r - \delta\Delta\varepsilon_a\right)}{-2} \tag{7}$$

where $\delta\Delta\varepsilon_a$ is the step change in axial strain; $\delta\Delta\sigma_r$ the step change in radial stress; $\delta\Delta\sigma_a$ the step change in axial stress (which is zero as the overburden is constant); and $\delta\Delta q$ is the step change in deviator stress.

The value of Secant Young's Modulus didn't require adjustment to implement this procedure. In the model the stress change is calculated and the strains are inputs. With this in mind the relationship between radial and axial strain change, which was found experimentally, had to be considered. The same logic as adopted for radial strain change was used:

$$\Delta\varepsilon_a = \sum_{1}^{n} \delta\Delta\varepsilon_a \tag{8}$$

where $\Delta\varepsilon_a$ the total change in axial strain; $\delta\Delta\varepsilon_a$ is step change in axial strain; and n the number of steps considered. To the step change in axial strain the value of $\Delta\varepsilon_a$ was calculated based on $\Delta\varepsilon_r$ and the hardening parameter for timestep n. This value was then held by the model and the process repeated for timestep n+1. This allowed the step change in axial strain to be calculated as:

$$\delta\Delta\varepsilon_a = \Delta\varepsilon_{a(n+1)} - \Delta\varepsilon_{a(n)} \tag{9}$$

Where again $\Delta\varepsilon_a$ is total change in axial strain; $\delta\Delta\varepsilon_a$ is step change in axial strain; and n the number of steps considered.

This adaptation to the model made it possible to implement the mathematical model using the timestep concept.

VALIDATION BY TRIAXIAL TEST – COMMERCIAL SPREADSHEET

The first stage of the validation process was to use a commercially available spreadsheet package, in this case Microsoft Excel, to ensure that the calculated values were comparable with the values observed experimentally. Initially the model was used to predict the density change of the sample under cycling. This was done for the three tests carried out by Xu [2005], using the same initial densities and change in radial strain range applied. The results were then compared to those of Xu and found to be accurate. This was essential as this is also used to obtain the hardening parameter on which many of the model parameters rely. Figure 3 shows a typical example.

The second stage of the Excel validation was to look at the individual elements. Predictions of Young's modulus and total change in axial strain against total change in radial strain were produced and compared to the experimental values of Xu. Again the model predictions, when compared to the experimental data, proved to be good.

These predictions for individual parameters allowed the stress calculation to be considered. Predicting the onset of failure and programming repeated cycles within a spreadsheet would be complex and labour intensive. Therefore a simplified method was used for both issues. Rather than predicting failure using Critical State Soil Mechanics an approach with bounds for passive and active failure was adopted. These were calculated based on the estimated angle of internal friction published by Xu. In the case of cycles only individual cycles were considered. Rather than basing the density and hardening parameter on the experimental values they were instead based on the predicted value. This allowed cycling to be tested without considering the stress calculation for every cycle. This was was carried out for a number of cycles, varying the radial strain range and the initial density.

Similarly to the FDM, the timestep concept was adopted. In the tests 30 steps were used. However, to ensure that limiting the number of timesteps in this way had no bearing on the outcome, a special test was carried out with 1000 time steps.

The lateral stress test was carried out for a number of cycles, varying the radial strain range and the initial density. It was found that the mathematical model performed well and gave results which were reasonable when compared to those observed by Xu 2005. Figure 4 shows a typical comparison between the predicted and experimental values. This showed that the model behaved as expected and was suitable for programming into the Finite Difference program FLAC.

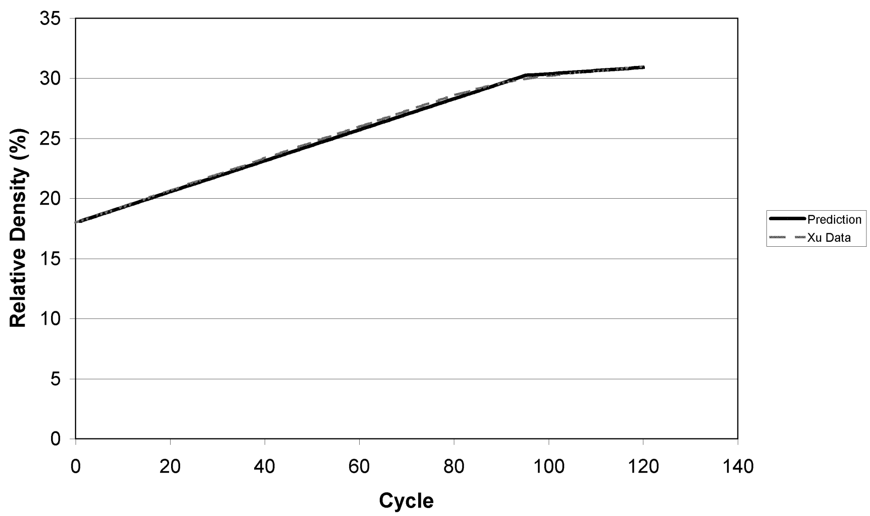

FIGURE 3 – COMPARISON OF MODEL PREDICTION AND EXPERIMENTAL DENSIFICATION

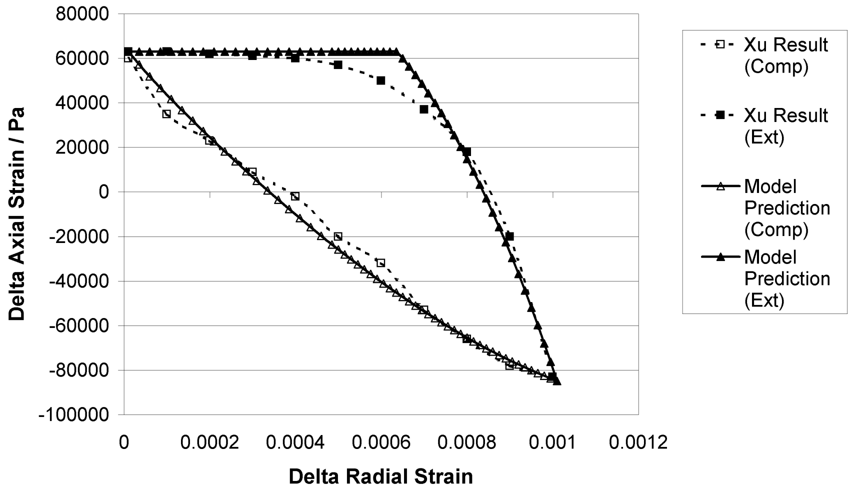

FIGURE 4 - TYPICAL CURVE OF LATERAL STRESS AND AGAINST RADIAL STRAIN FROM GRANULAR MODEL DEVELOPED IN MICROSOFT EXCEL

VALIDATION BY TRIAXIAL TEST – FLAC

The model was implemented a finite difference package. The package FLAC by the Itasca Corporation was chosen as being suitable. FLAC has the built in programming pseudo-language Fish (FLAC-ish) making it possible to programme the mathematical model. The programme style was based on the existing models in FLAC such as Mohr-Coulomb and Cam Clay.

As discussed, failure was to be predicted using Critical State Soil Mechanics. The Cam Clay model uses this so the failure criterion was adopted. It was used in the form as implemented already in FLAC, due to a lack of experimental evidence to the contrary.

Triaxial Test Model

A model was created which simulated the triaxial test, an axi-symmetric model with a single element. The granular constitutive model was assigned to the element with the properties as found by Xu [2005]. The initial loading condition was then defined, with an overburden of 80kPa to simulate a typical element at 4m depth, and at rest horizontal stresses.

A loop was created to apply the compressive change in radial strain being considered, have a rest period with no movement, apply the extension change in radial strain and have a second rest period. Each stage was applied over 1000 timesteps with the velocity set to achieve the change in radial strain desired. The vertical pressure was held constant at 80kPa and the loop repeated to achieve the number of radial strain change cycles required.

Initially the granular model was run with the reduced triaxial form of Hooke's Law given in (7). Later this was changed to the form of Hooke's Law given in (6).

Results

A similar piecemeal approach to testing was applied to the FLAC case as had been adopted with the spreadsheet. The triaxial model was tested using the in-built elastic model to ensure that it ran as expected. The elastic model was then replaced by the granular model using (7) to calculate the lateral stress change. Each aspect of the model was tested in turn.

The first aspect to be tested was the radial strain input. This was controlled both by the granular model and the triaxial test model. Figure 5 shows the radial strain input acting as expected, negative strain change denotes compression in accordance with FLAC sign convention.

Many of the essential parameters rely on the Hardening Parameter, γ. Therefore the first part to be checked was the development of the Hardening Parameter with applied cycles of radial strain change. This was found to work well and allowed the various coefficients reliant on this to be checked. Individual variables such as secant horizontal Young's modulus and the change in axial strain were then tested to ensure that they worked properly.

The final stage was to run the model calculating the stress changes. A typical plot of change in radial strain against deviator stress over many cycles is shown in Figure 6, which may be compared with the corresponding experimental results by Xu [2005] given in Figure 1. The model behaves as expected, with peak lateral stress increaseing with cycles and the active state met upon each unloading excursion. The model prediction is also accurate within an individual cycle meaning that part cycles can also be modelled.

Equation (7) was replaced by (6) in the granular model. The same test was rerun and the results found to be replicated. The final stage of the validation process tested the failure criterion further by running the triaxial test until passive failure occurred.

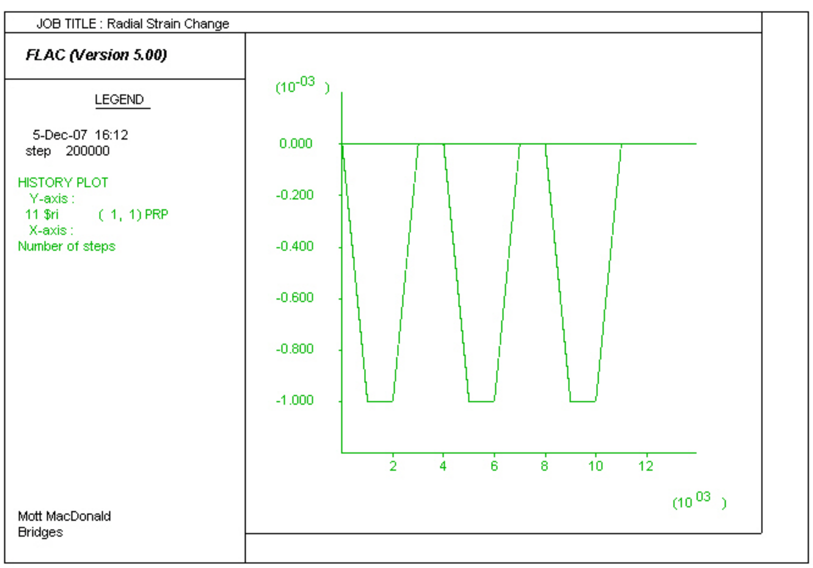

FIGURE 5 - TYPICAL CURVE OF RADIAL STRAIN CHANGE WITH TIMESTEPPING IN FDM

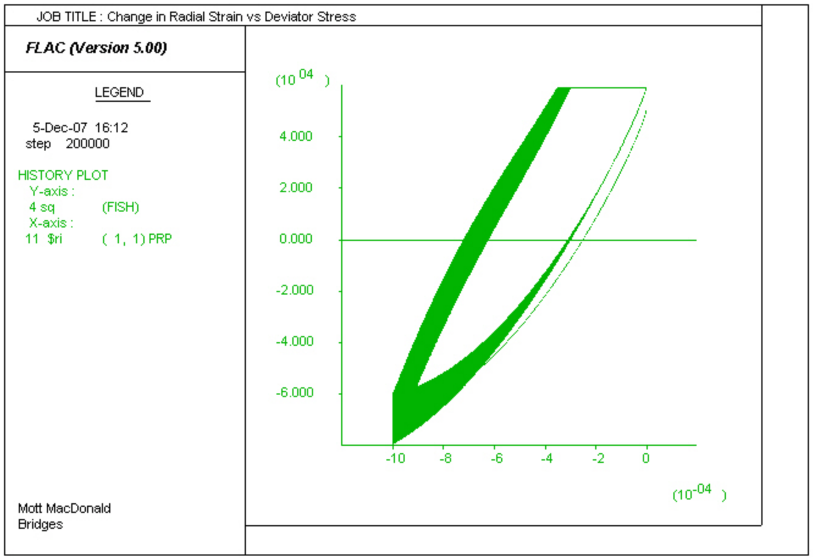

FIGURE 6 - TYPICAL CURVE OF LATERAL STRESS AND AGAINST RADIAL STRAIN OVER REPEATED STRAIN CYCLES

CONCLUSIONS

The advantages of using Integral Bridge construction has been widely publicised, but equally the disadvantages associated with the potential for soil pressures to rise during the life of the structure cannot be neglected. The work of Xu [2005] showed the fundamental behaviour associated with soil behind an integral bridge and subsequent modelling work used this to develop a granular soil model suitable for cyclic loading associated with an integral bridge.

This paper has discussed the basis of this model and the subsequent validation work that aimed to ensure the model captured essential elements of the soil behaviour. First testing the mathematical model as fully as practicably possible in a commercial spreadsheet ensured that the programme basis was sound. When the model was then programmed and tested in a Finite Difference program, it gave predictions which compared favourably to the experimental results found by Xu.

This shows that the model works well for a single element. The next research stage will be to extend the model from a single element to an entire integral bridge soil-structure system. Research by others on the overall abutment system behaviour, such as that by Tapper and Lehane [2004], will be used to validate the theory that this behaviour is indicative of all element behaviour behind an abutment. This could lead to a powerful model, suitable in both the design and research of soil-structure interaction occurring with an integral bridge. This could in turn be used to achieve both efficiency in design and construction.

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